Building up an Annuity

Lecture 4

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Outline

Definitions

Definition (Annuity (Webster))

An annuity is a plan where a fixed amount of money that is paid in regular payments to a person over time.

- Typically, a retirement plan is an annuity – You invest over your working life and then withdraw from it monthly during retirement.
- One could establish an annuity to pay for a child's college education – You invest for 18 years and withdraw from it semiannually over the next 4 years.

Annuities

- An annuity has two stages.
  - The investment stage (building it up).
  - The withdrawal stage (drawing it down).
- During the investment stage, the balance grows.
- During the withdrawal stage, the balance diminishes.

Annuities

- Google “beware of annuities.”
  - (Forbes) Annuities: The Good, the Bad, and The Ugly
- These annuities are products sold by insurance companies–there are pitfalls.

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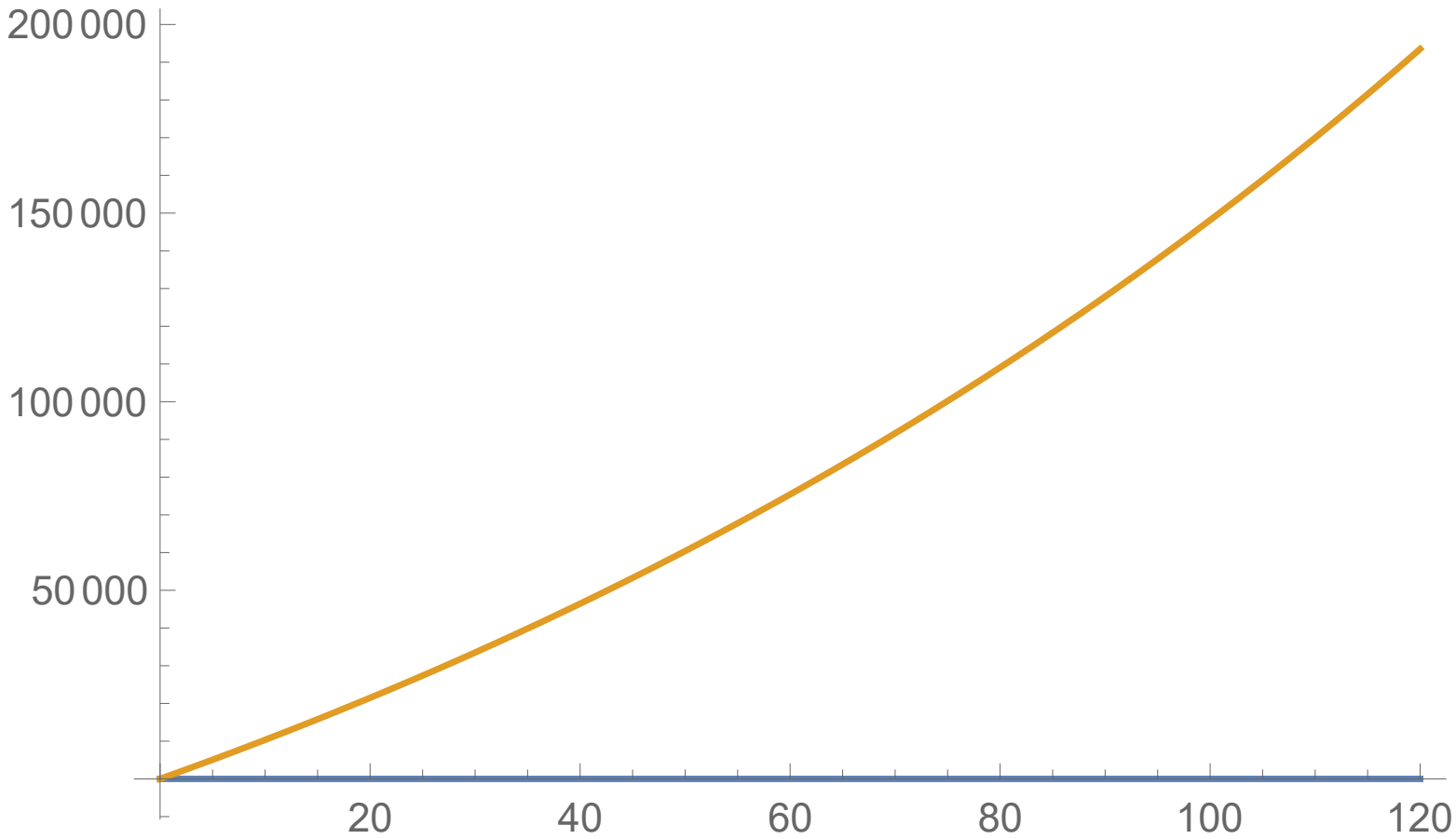
- These annuities are products sold by insurance companies–there are pitfalls.
- Be a fan of the NFL.
- NFL = No Free Lunch.
- We are focused only on the *concept* of an annuity, not the commercial product.

Outline

- Twenty years ago, the DJIA was at about 830.
- Now it is at about 26000.
- That represents an annual growth rate of 9%.

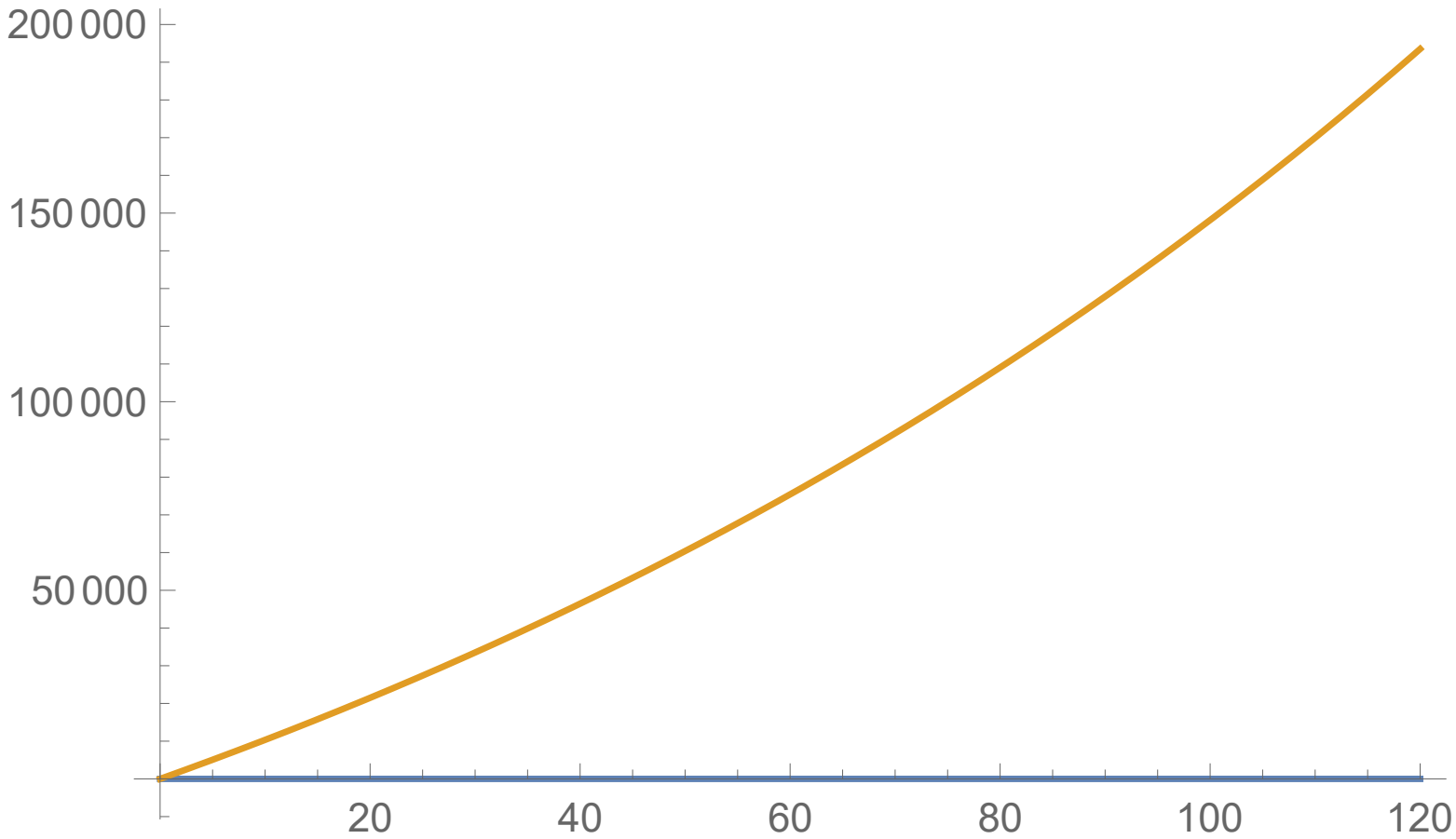
The Effect of Time



Invest $1,000 per month at 9% for 10 years

Save up $193,514.28.

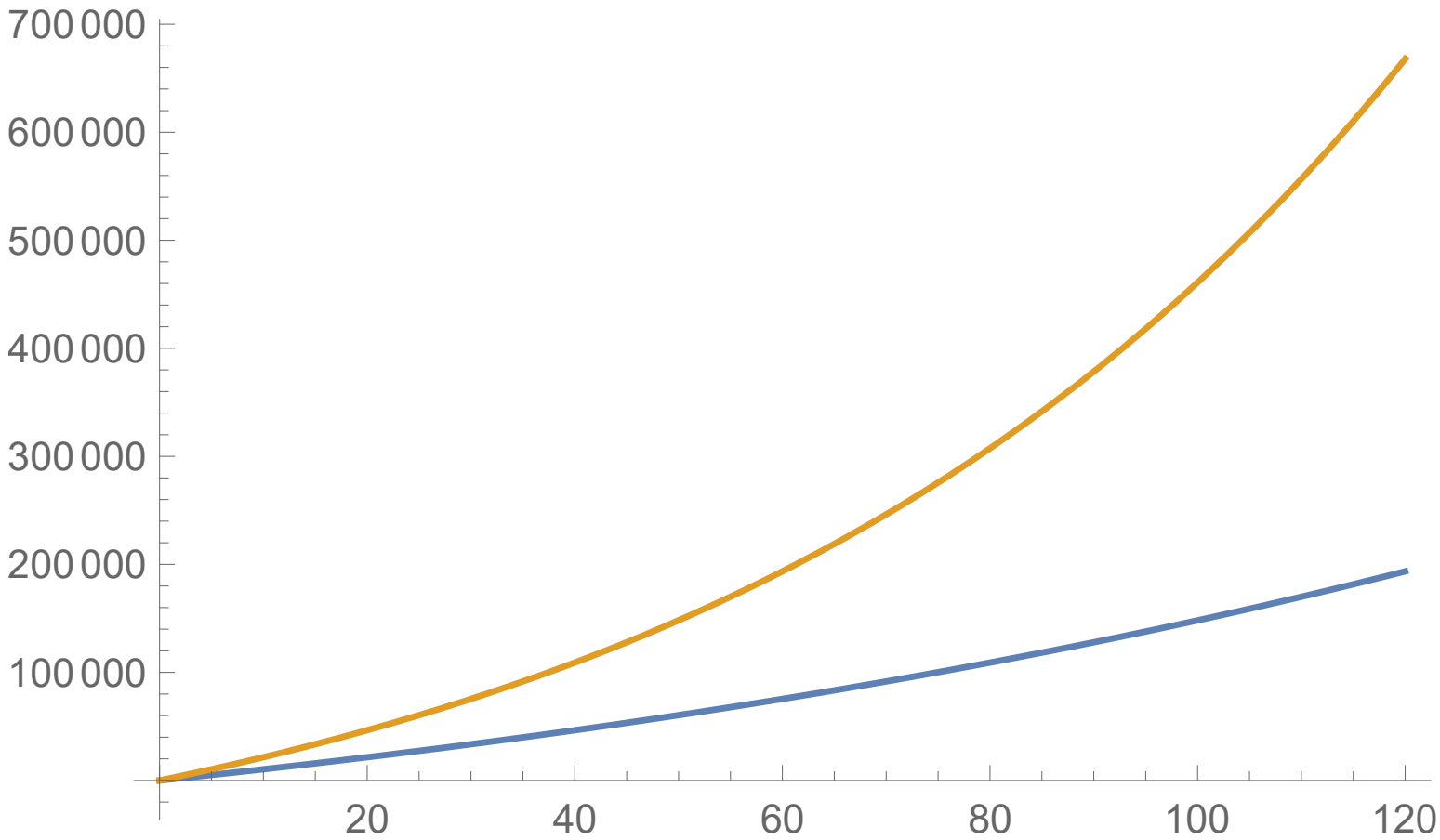
The Effect of Time



Invest $1,000 per month at 9% for 10 years

Withdraw $1,741.10 per month for 20 years

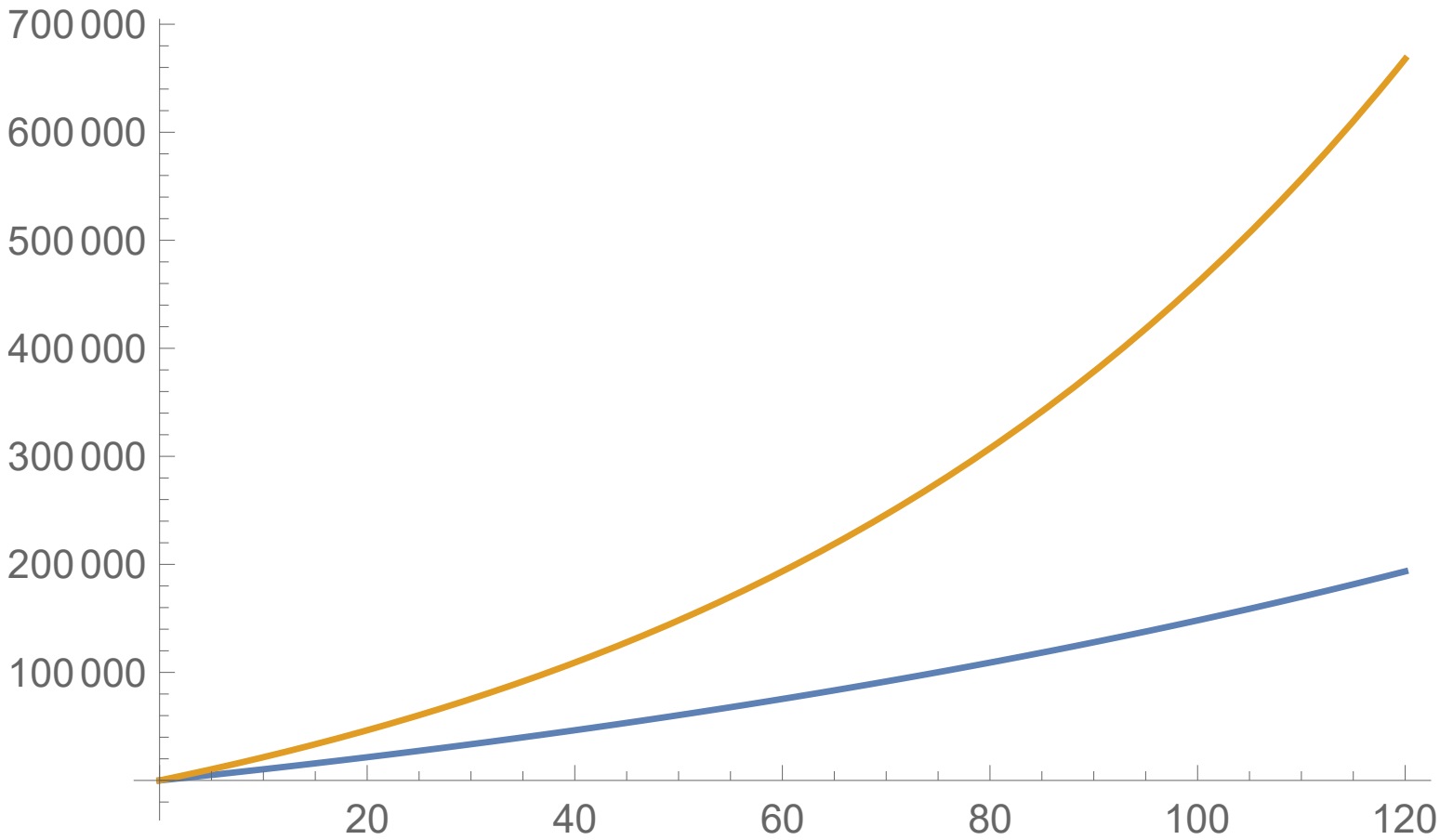
The Effect of Time



Invest $1,000 per month at 9% for 20 years

Save up $667,886.87.

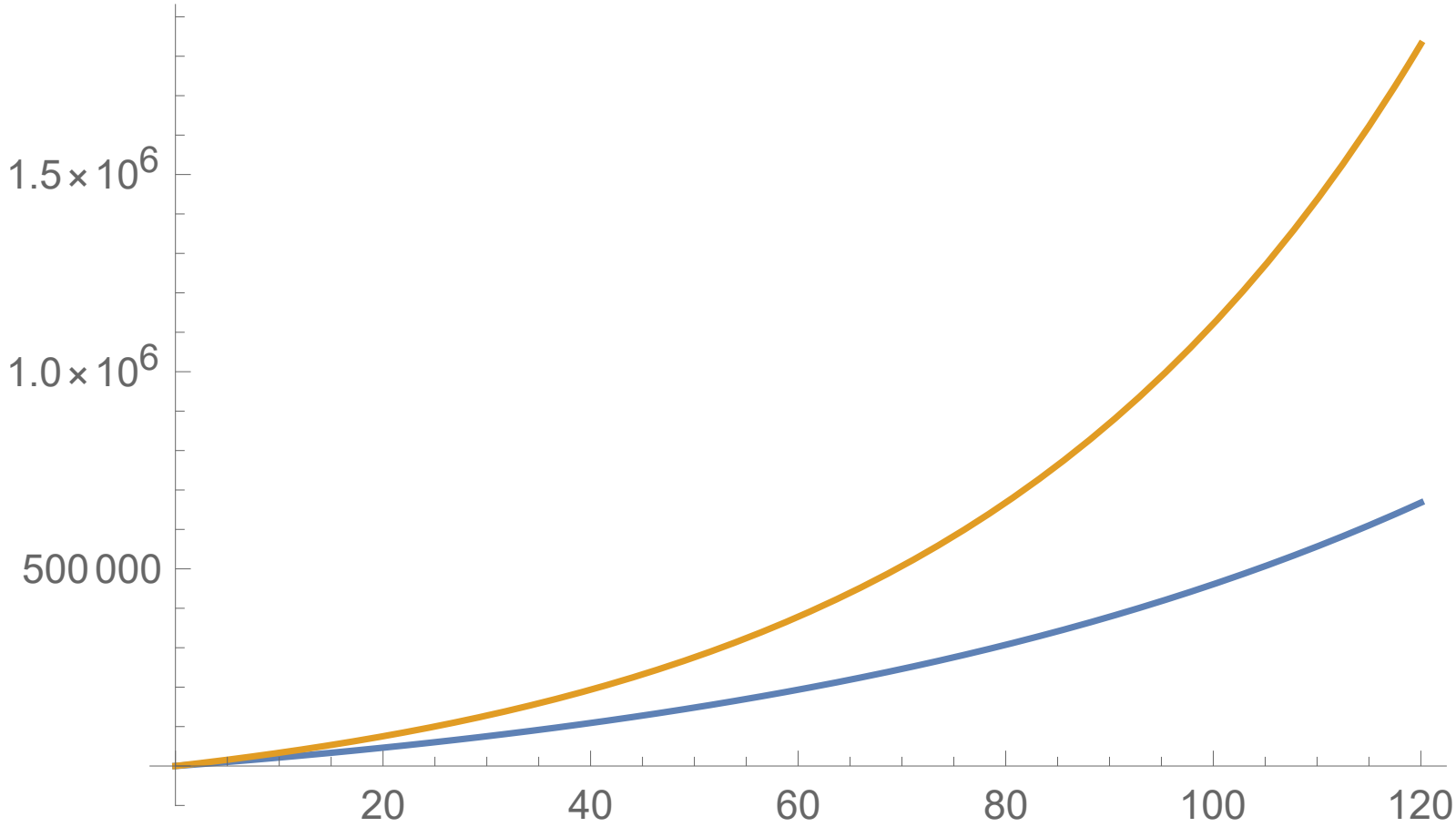
The Effect of Time



Invest $1,000 per month at 9% for 20 years

Withdraw $6,009.15 per month for 20 years

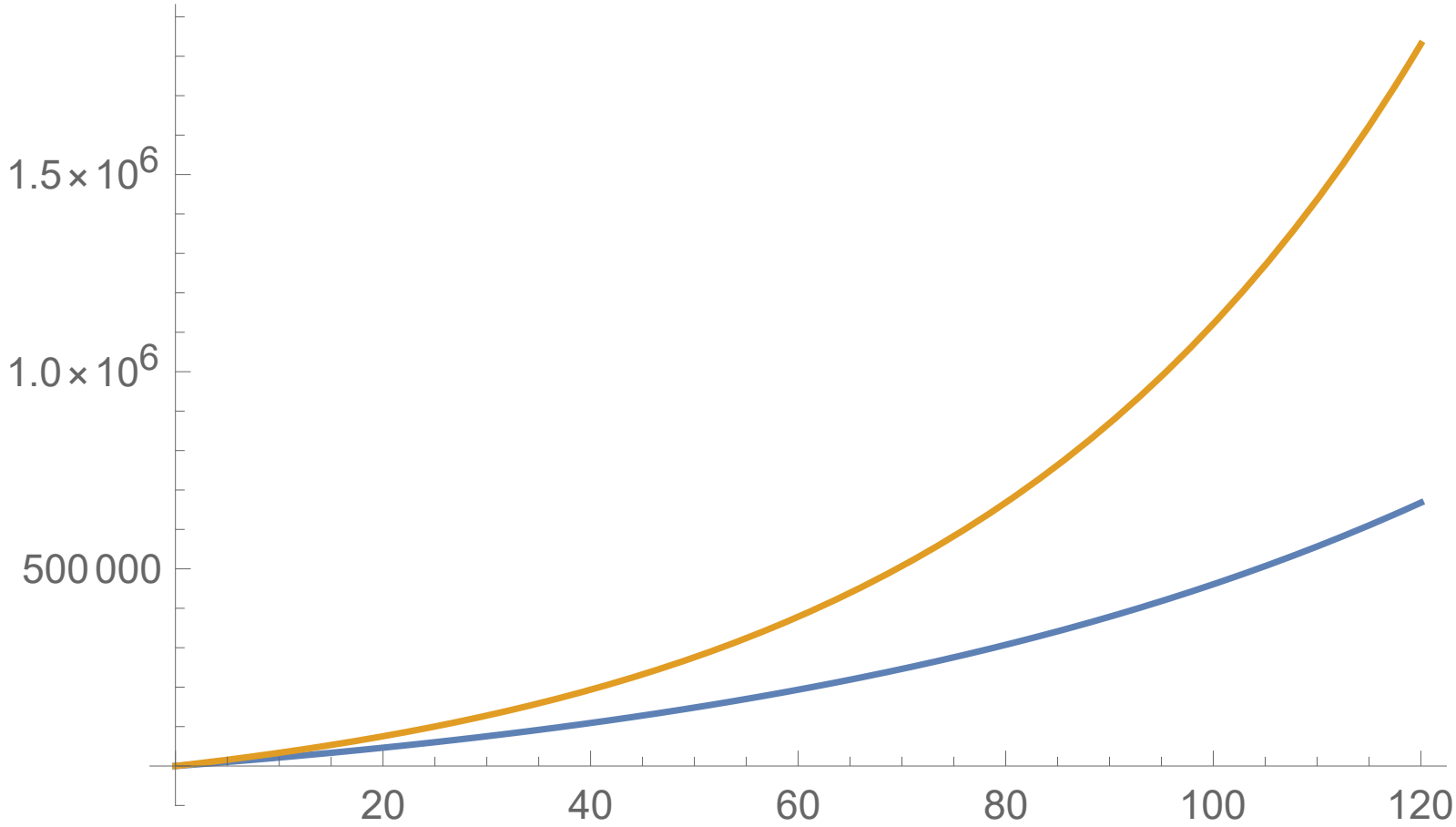
The Effect of Time



Invest $1,000 per month at 9% for 30 years

Save up $1,830,743.48.

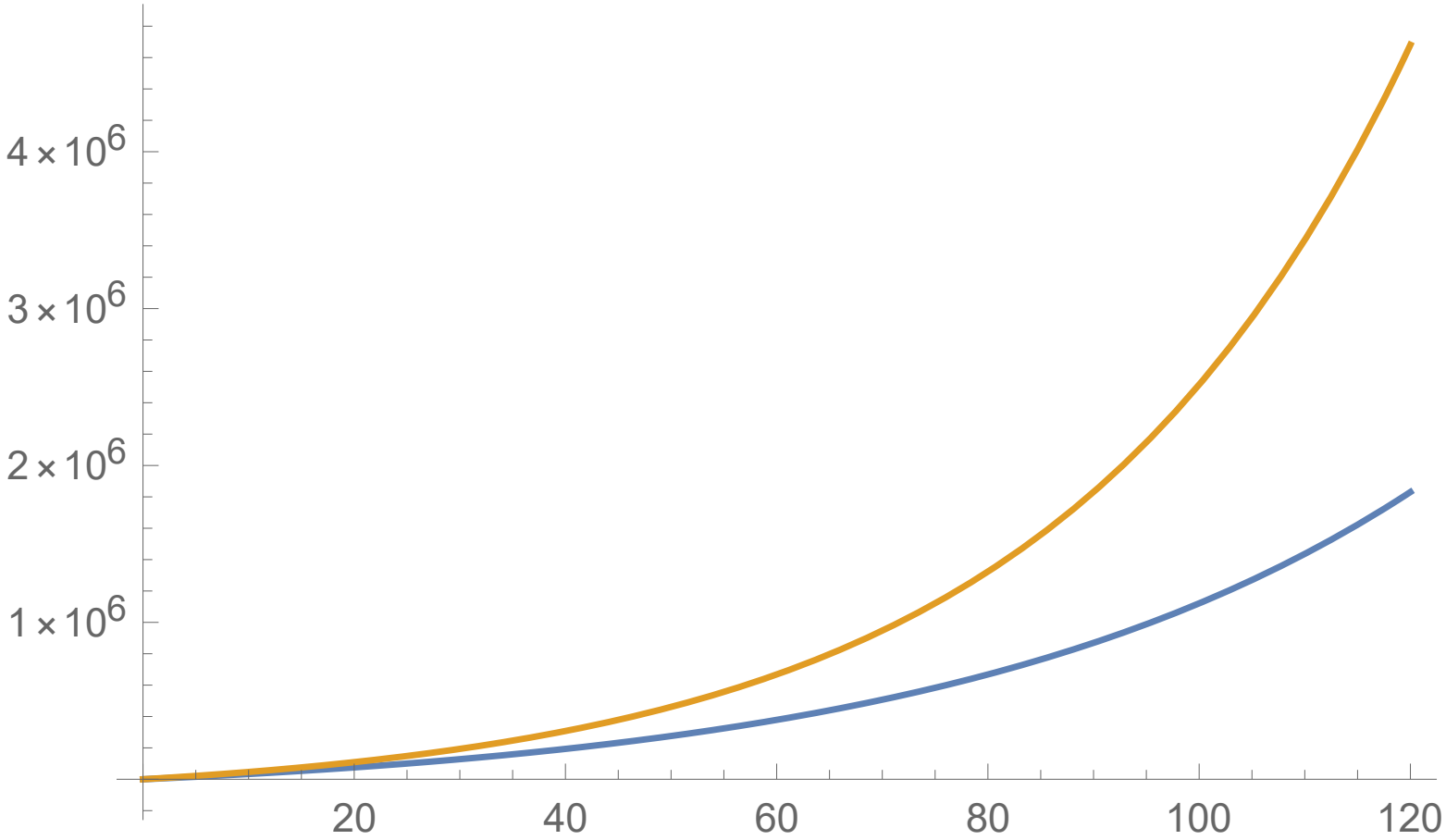
The Effect of Time



Invest $1,000 per month at 9% for 30 years

Withdraw $16,471.67 per month for 20 years

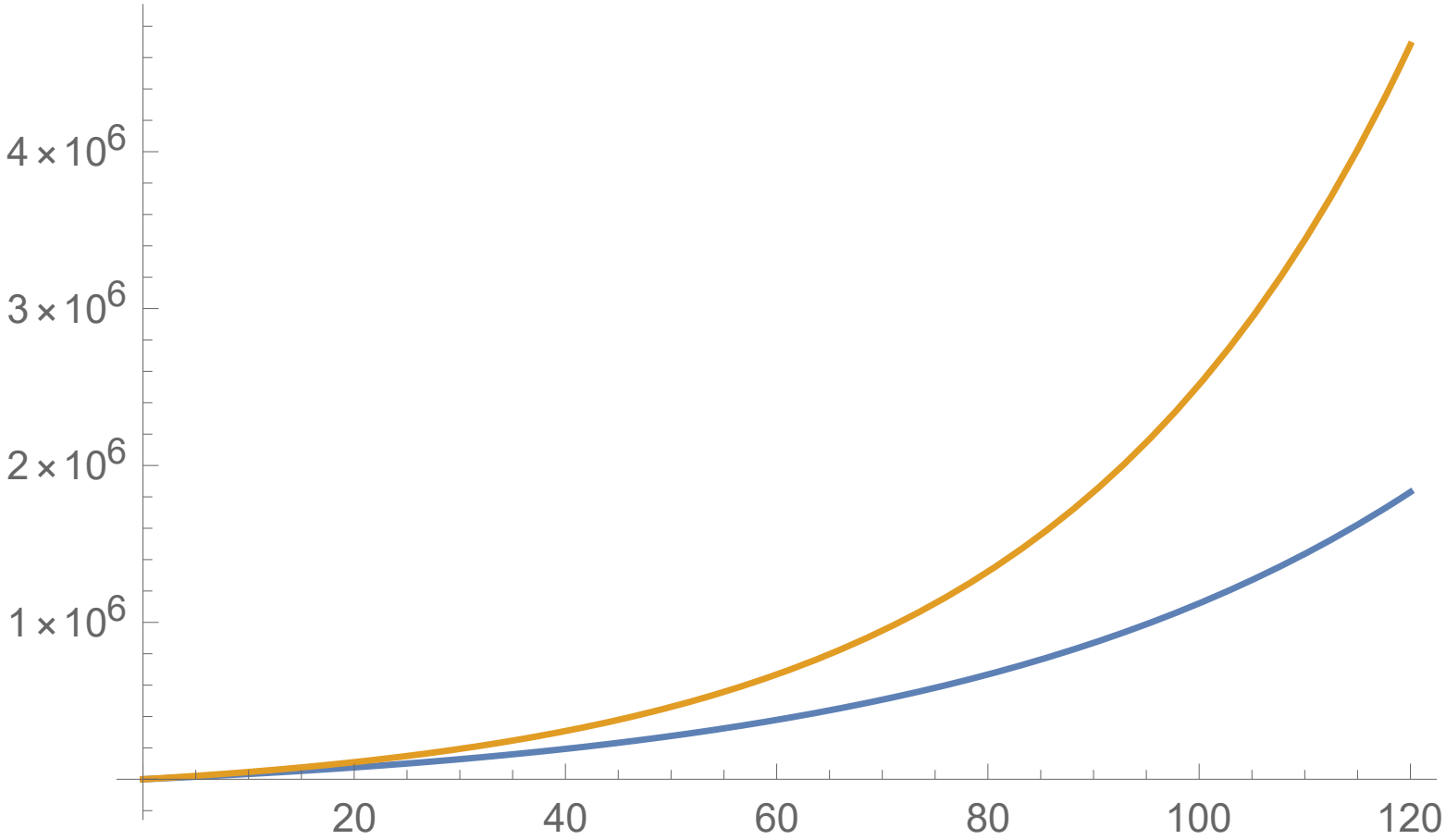
The Effect of Time



Invest \$1,000 per month at 9% for 40 years

Save up \$4,681,320.27.

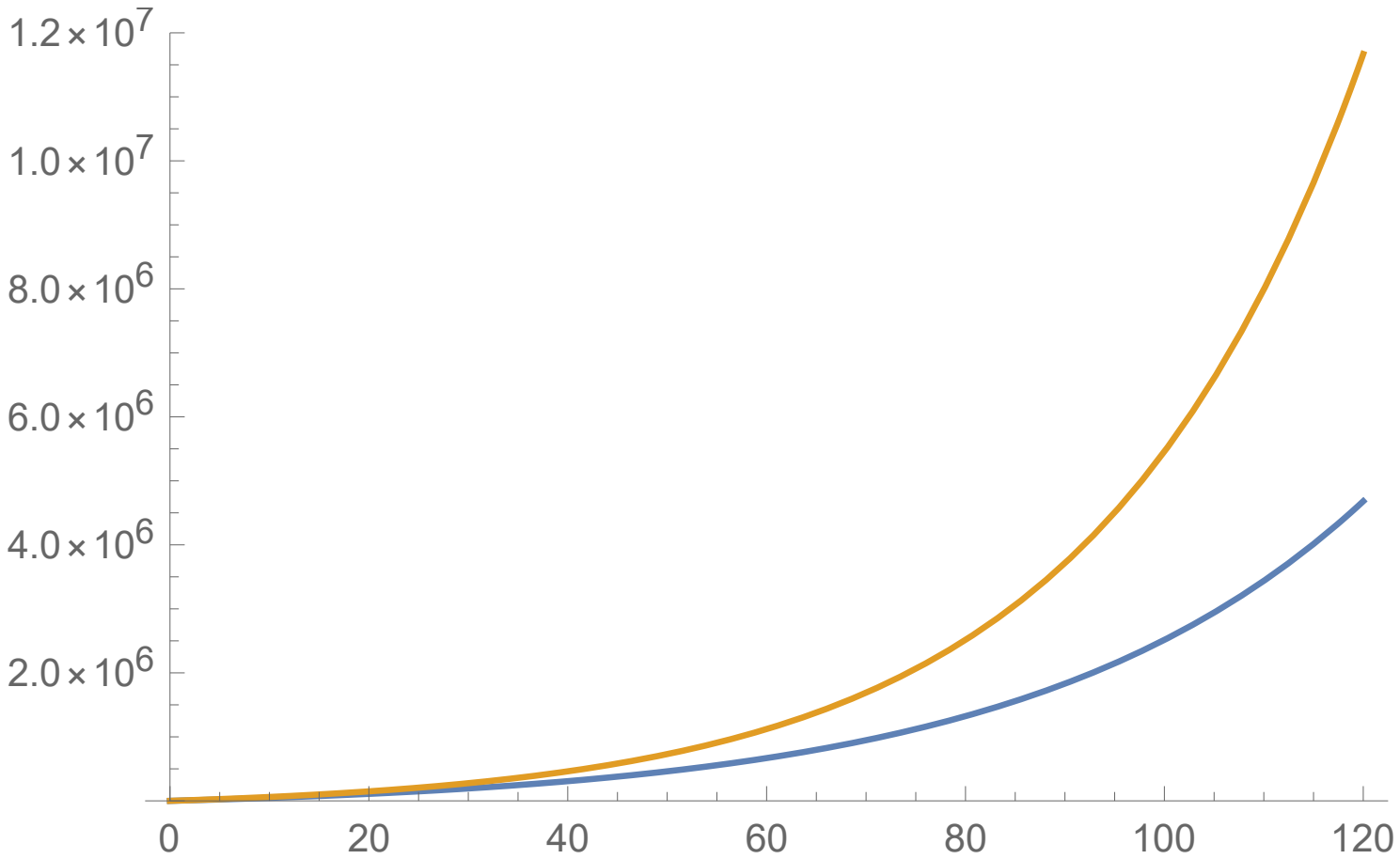
The Effect of Time



Invest \$1,000 per month at 9% for 40 years

Withdraw \$42,119.05 per month for 20 years

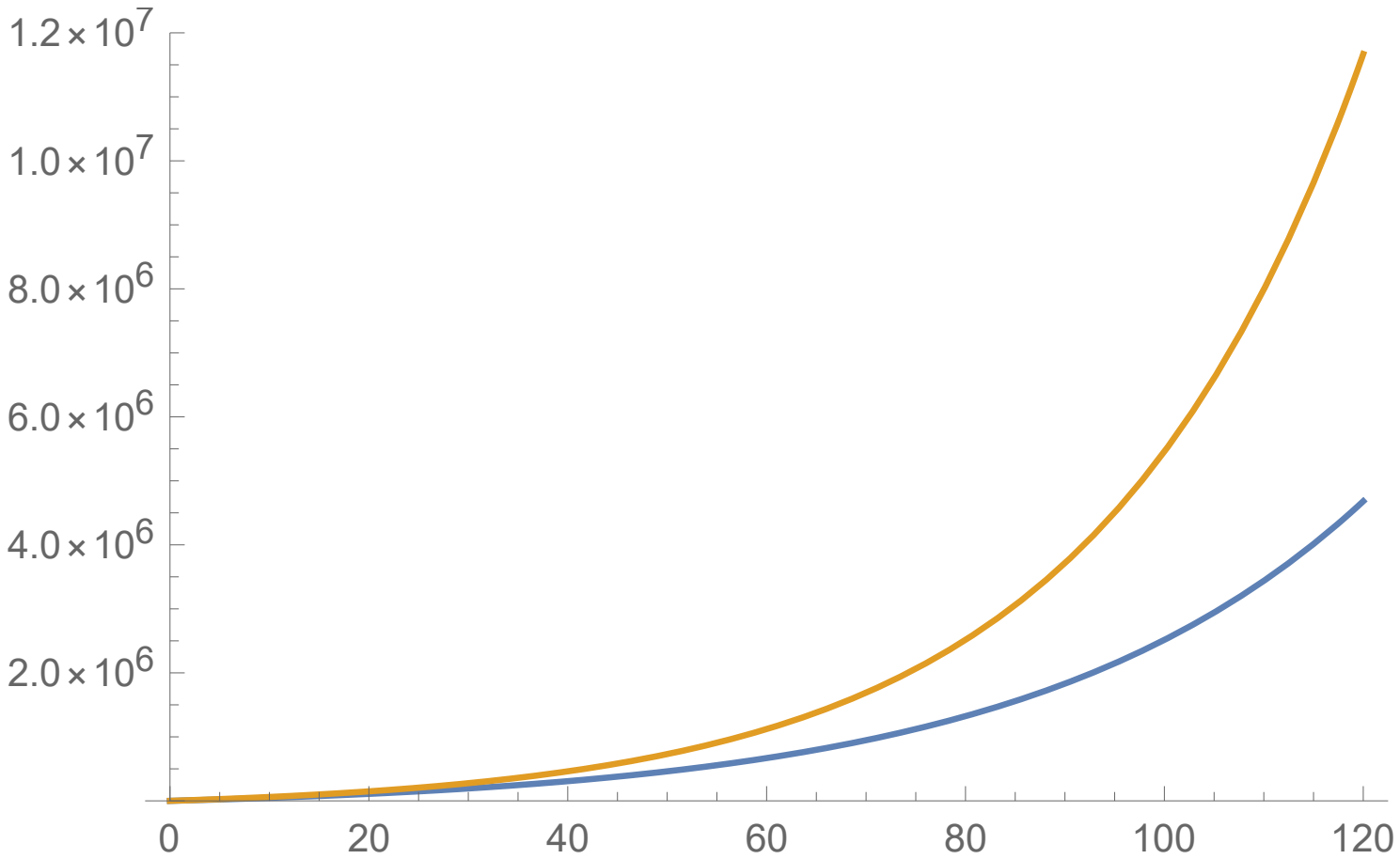
The Effect of Time



Invest $1,000 per month at 9% for 50 years

Save up $11,669,101.86.

The Effect of Time



Invest \$1,000 per month at 9% for 50 years

Withdraw \$104,989.94 per month for 20 years

Outline

Example

Example (Three Deposits)

- Let the annual interest rate be 10%.
- Invest $1000 each year for 5 years.

Example

Example (Three Deposits)

The investment stage:

Year	Starting Balance	Interest	Total	Deposit	Ending Balance
1	0.00	0.00	0.00	1,000.00	1,000.00

Example

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The investment stage:

Year	Starting Balance	Interest	Total	Deposit	Ending Balance
1	0.00	0.00	0.00	1,000.00	1,000.00
2	1,000.00	100.00	1,100.00	1,000.00	2,100.00

Example

Example (Three Deposits)

The investment stage:

Year	Starting Balance	Interest	Total	Deposit	Ending Balance
1	0.00	0.00	0.00	1,000.00	1,000.00
2	1,000.00	100.00	1,100.00	1,000.00	2,100.00
3	2,100.00	210.00	2,310.00	1,000.00	3,310.00

Example

Example (Three Deposits)

The investment stage:

Year	Starting Balance	Interest	Total	Deposit	Ending Balance
1	0.00	0.00	0.00	1,000.00	1,000.00
2	1,000.00	100.00	1,100.00	1,000.00	2,100.00
3	2,100.00	210.00	2,310.00	1,000.00	3,310.00
4	3,310.00	331.00	3,641.00	1,000.00	4,641.00

Example

Example (Three Deposits)

The investment stage:

Year	Starting Balance	Interest	Total	Deposit	Ending Balance
1	0.00	0.00	0.00	1,000.00	1,000.00
2	1,000.00	100.00	1,100.00	1,000.00	2,100.00
3	2,100.00	210.00	2,310.00	1,000.00	3,310.00
4	3,310.00	331.00	3,641.00	1,000.00	4,641.00
5	4,641.00	464.10	5,105.10	1,000.00	6,105.10

Annuity Formula (Building Up)

- If the payments are annual, then $k = 1$ and the formula is not too bad:

$$F = P\left(\frac{(1+r)^t - 1}{r}\right)$$

where F is the future value of the annuity, P is the amount investment per period, r is the annual interest rate, and t is the number of years.

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- When k is greater than one, then the formula is a bit more complicated.

$$F = P\left(\frac{(1+\frac{r}{k})^{kt} - 1}{r/k}\right).$$

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  where F is the future value of the annuity, P is the amount investment per period, r is the annual interest rate, and t is the number of years.
- When k is greater than one, then the formula is a bit more complicated.

$$F = P\left(\frac{(1+\frac{r}{k})^{kt} - 1}{r/k}\right).$$

- We replace r with r/k and t with kt.

Example

Example (Three Deposits)

The future value is

$$F = \frac{P\left((1+r)^t - 1\right)}{r}$$

Example

Example (Three Deposits)

The future value is

$$\begin{aligned} F &= \frac{P\left((1+r)^t - 1\right)}{r} \\ &= \frac{1000((1.10)^5 - 1)}{0.10} \end{aligned}$$

Example

Example (Three Deposits)

The future value is

$$\begin{aligned} F &= \frac{P\left((1+r)^t - 1\right)}{r} \\ &= \frac{1000((1.10)^5 - 1)}{0.10} \\ &= \$6,105.10. \end{aligned}$$

Outline

Another Example

Example

- A person earning $48,000 a year invests 5% of his income in a retirement account earning 8% per year for 45 years.
- How much does he have at the end of 45 years (annual payments)?

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- A person earning $48,000 a year invests 5% of his income in a retirement account earning 8% per year for 45 years.
- How much does he have at the end of 45 years (annual payments)? ans: $927,613.48
- How much does he have at the end of 45 years (monthly payments)?

Another Example

Example

- A person earning $48,000 a year invests 5% of his income in a retirement account earning 8% per year for 45 years.
- How much does he have at the end of 45 years (annual payments)? ans: $927,613.48
- How much does he have at the end of 45 years (monthly payments)? ans: $1,054,907.98

Another Example

Example

- A person earning $48,000 a year invests 5% of his income in a retirement account earning 8% per year for 45 years.
- How much does he have at the end of 45 years (annual payments)? ans: $927,613.48
- How much does he have at the end of 45 years (monthly payments)? ans: $1,054,907.98
- What if the account earned 9% APR (monthly payments)?

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Example

- A person earning $48,000 a year invests 5% of his income in a retirement account earning 8% per year for 45 years.
- How much does he have at the end of 45 years (annual payments)? ans: $927,613.48
- How much does he have at the end of 45 years (monthly payments)? ans: $1,054,907.98
- What if the account earned 9% APR (monthly payments)? ans: $1,480,975.69

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Example

- A person earning $48,000 a year invests 5% of his income in a retirement account earning 8% per year for 45 years.
- How much does he have at the end of 45 years (annual payments)? ans: $927,613.48
- How much does he have at the end of 45 years (monthly payments)? ans: $1,054,907.98
- What if the account earned 9% APR (monthly payments)? ans: $1,480,975.69
- What if the account earned 12% APR (monthly payments)?

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Example

- A person earning $48,000 a year invests 5% of his income in a retirement account earning 8% per year for 45 years.
- How much does he have at the end of 45 years (annual payments)? ans: $927,613.48
- How much does he have at the end of 45 years (monthly payments)? ans: $1,054,907.98
- What if the account earned 9% APR (monthly payments)? ans: $1,480,975.69
- What if the account earned 12% APR (monthly payments)? ans: $4,290,938.61

Outline

Assignment

Assignment

- Annuity worksheet: 1 - 5.